Bayesian nonparametric modeling for comparison of neuronal intensity firing rates

Valerie Poynor vpoynor@soe.ucsc.edu University of California, Santa Cruz

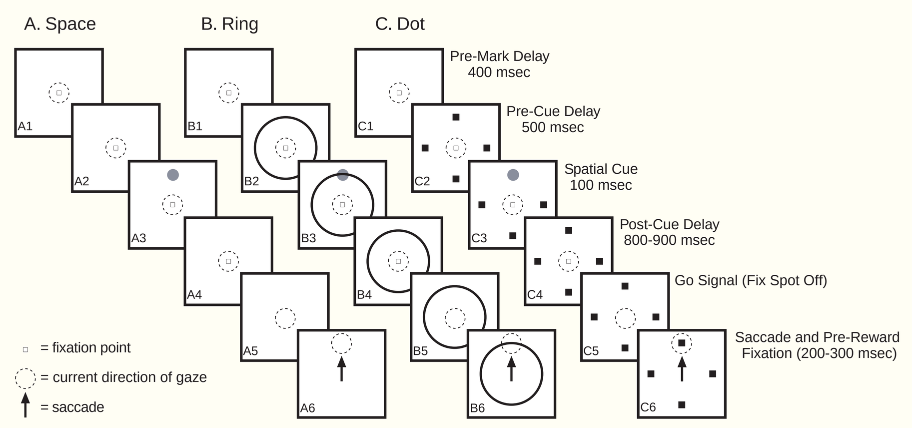
Athanasios Kottas, University of California, Santa Cruz Sam Behseta, California State University, Fullertion David E. Moorman, University of South Carolina, Charleston Carl R. Olson, Carnegie Mellon University

Abstract

In neuroscience, there is great interest in comparing neuronal firing rates across multiple conditions. Traditionally, neuroscientists record the firing activity of neurons under these conditions for a number of trials. In this research, we are comparing neuronal activity obtained from the Supplementary Eye Field (SEF) area of a macaque monkey's brain responding to three different visual stimuli. Data were recorded for 4000 milliseconds per condition per trial. We propose a flexible Bayesian nonparametric dependent Dirichlet process (DDP) mixture model to jointly model the nonhomogeneous Poisson process (NHPP) intensity function of the neuronal firing times across the three conditions. Under these modeling techniques we are able to borrow strength from conditions having higher firing rates to make inference on conditions where the neuron exhibits less activity, all the while maintaining a data driven distributional structure. We illustrate the methodology with comparison of an SEF neuron firing rates across the three conditions.

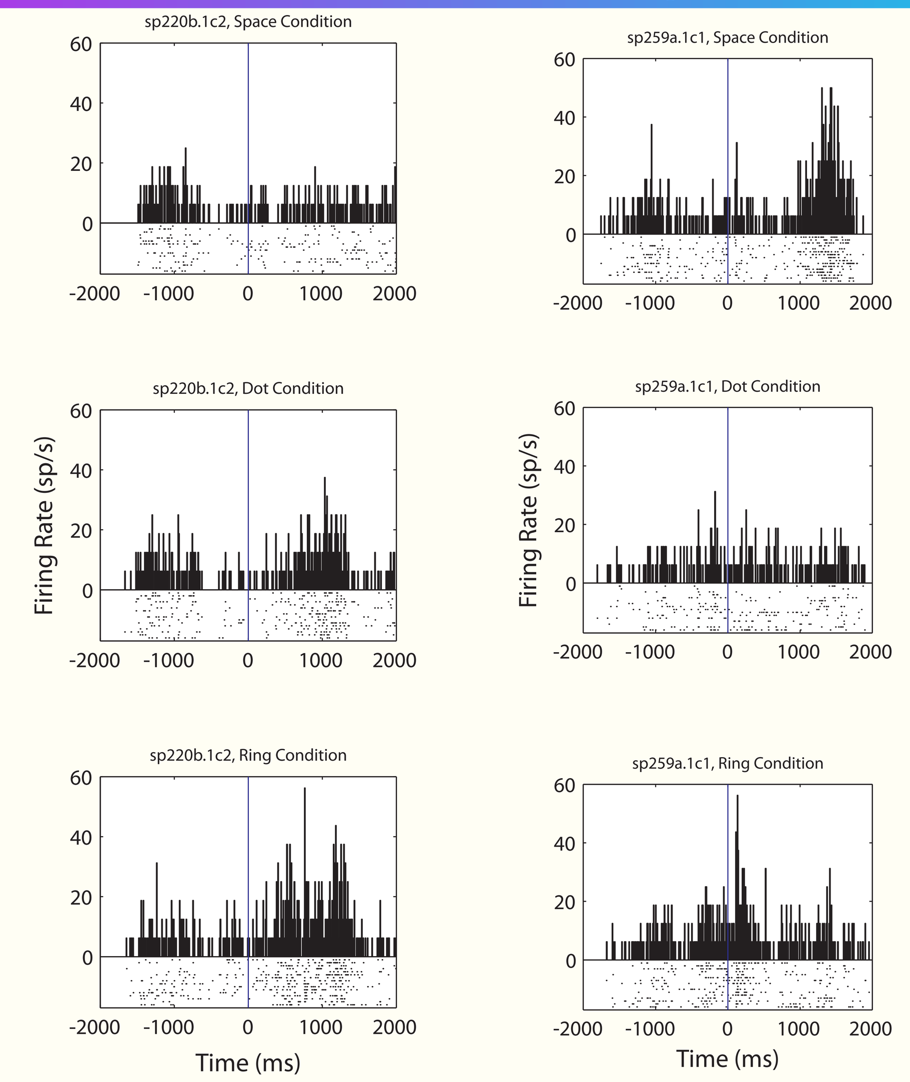
Experiment

The SEF region of a primate brain is involved in the planning and control of saccadic eye movements. The data we use here are a subset of a much larger experiment designed to investigate neuronal firing characteristics under sensory, cognitive, and behavioral factors.



We will focus on two particular neurons recorded for a 4000ms (−2000ms, 2000ms) time interval for three conditions in which a memory-guided saccade was made to a visual target (space, ring, or dot). There were 16 trials performed under each condition.

Peri-Stimulus Time Historgrams



Model

Let $i = 1, 2, 3$ represent the Space, Dot, and Ring conditions respectively. Let $t_{\cdot jk} = (t_{1jk}, t_{2jk}, t_{3jk})'$ be the triplet vector of the k^{th} ($k = 1, ..., n_j$) firing time in a particular trial, j, for $j = 1, ..., 16$. We are interested in modeling the intensity functions, $\boldsymbol{\lambda}_i$, for $i = 1, 2, 3$, of the NHPP. The likelihood for all firing times $t_{\cdot jk}$ for $k = 1, ..., n_j$ and $j = 1, ..., 16$ is given by

$$\left[\prod_{i=1}^{3}\left(exp\left\{-\int_0^1 \boldsymbol{\lambda}_i(u)du\right\}\prod_{j=1}^{16}\prod_{k=1}^{n_j}\boldsymbol{\lambda}_i(t_{ijk})\right)\right]^{s_{ijk}}$$

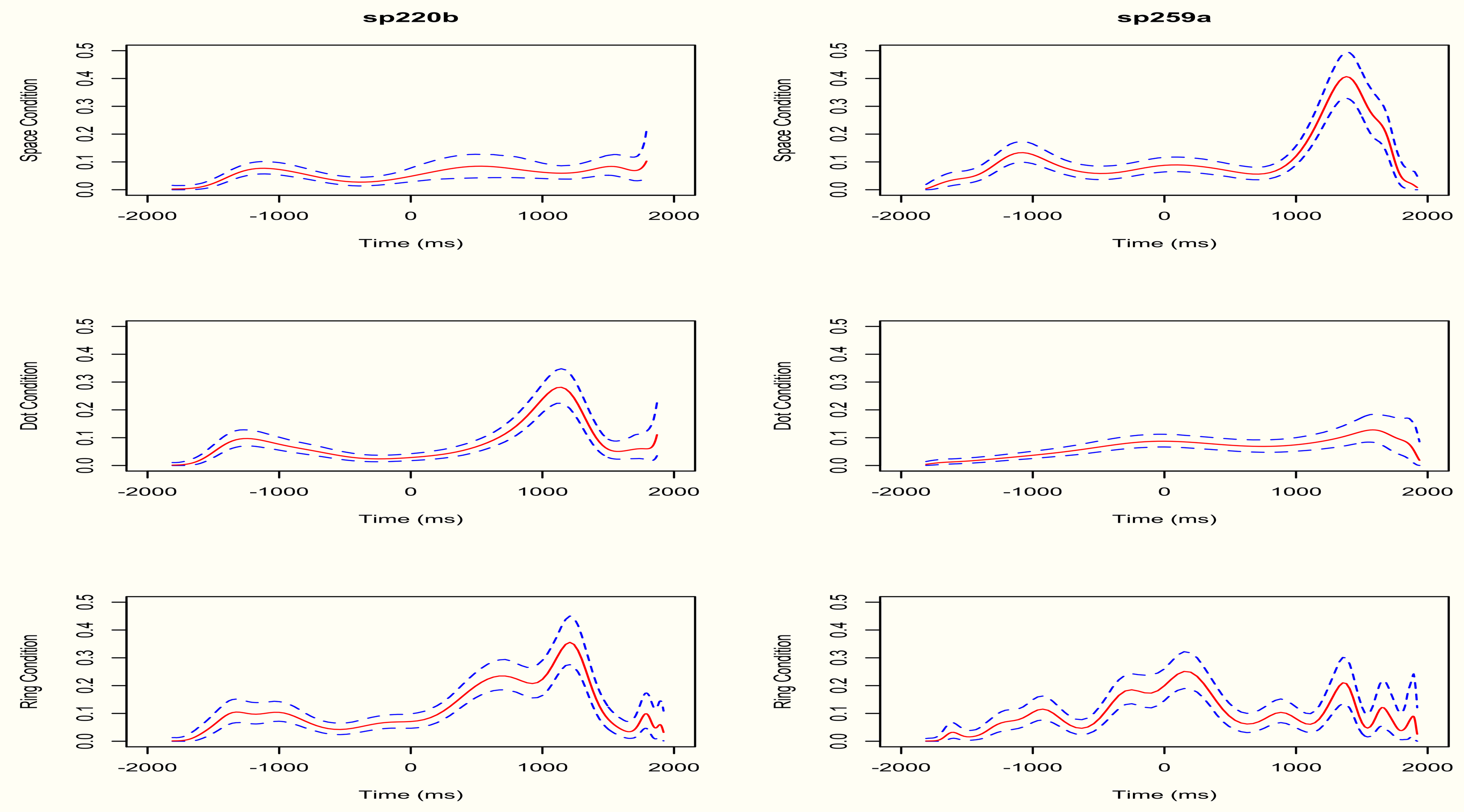
where $s_{ijk} = 1$ if t_{ijk} is observed and 0 otherwise. We use a nonparametric prior on $\{\boldsymbol{\lambda}_i : i = 1, 2, 3\}$ that incorporates the dependence of firing times across the conditions. We transform the data to the scale $(0, 1)$: $t \to y \in (0, 1)$, and model the density that defines the intensity function ($f_i(y) = \boldsymbol{\lambda}_i(y)/\gamma_i$ where $\gamma_i = \int_0^1 \boldsymbol{\lambda}_i(u)du$) using a logit-normal dependent Dirichlet process mixture model

$$\begin{aligned} y_{ijk}|\sigma_i^2, G_i &\sim \int \text{logit-}N(y_{ijk};\mu,\sigma_i^2)dG_i(\mu) \\ &= \sum_{l=1}^{L} p_l \text{logit-}N(y_{ijk};\mu_{il},\sigma_i^2) \end{aligned}$$

Marginally, $G_i \sim DP(\alpha, G_0)$, for $i = 1, 2, 3$, such that $\sum_{l=1}^{L} p_l \delta_{\theta_{li}}$ where the p_l's are the weights obtained via DP stick-breaking construction corresponding to the component $\theta_{li} = (\mu_{li})$ and L is the total number of components specified in the model. The dependence across the conditions are implied through the common locations $\theta_{l\cdot} = (\theta_{l1}, \theta_{l2}, \theta_{l3})$.

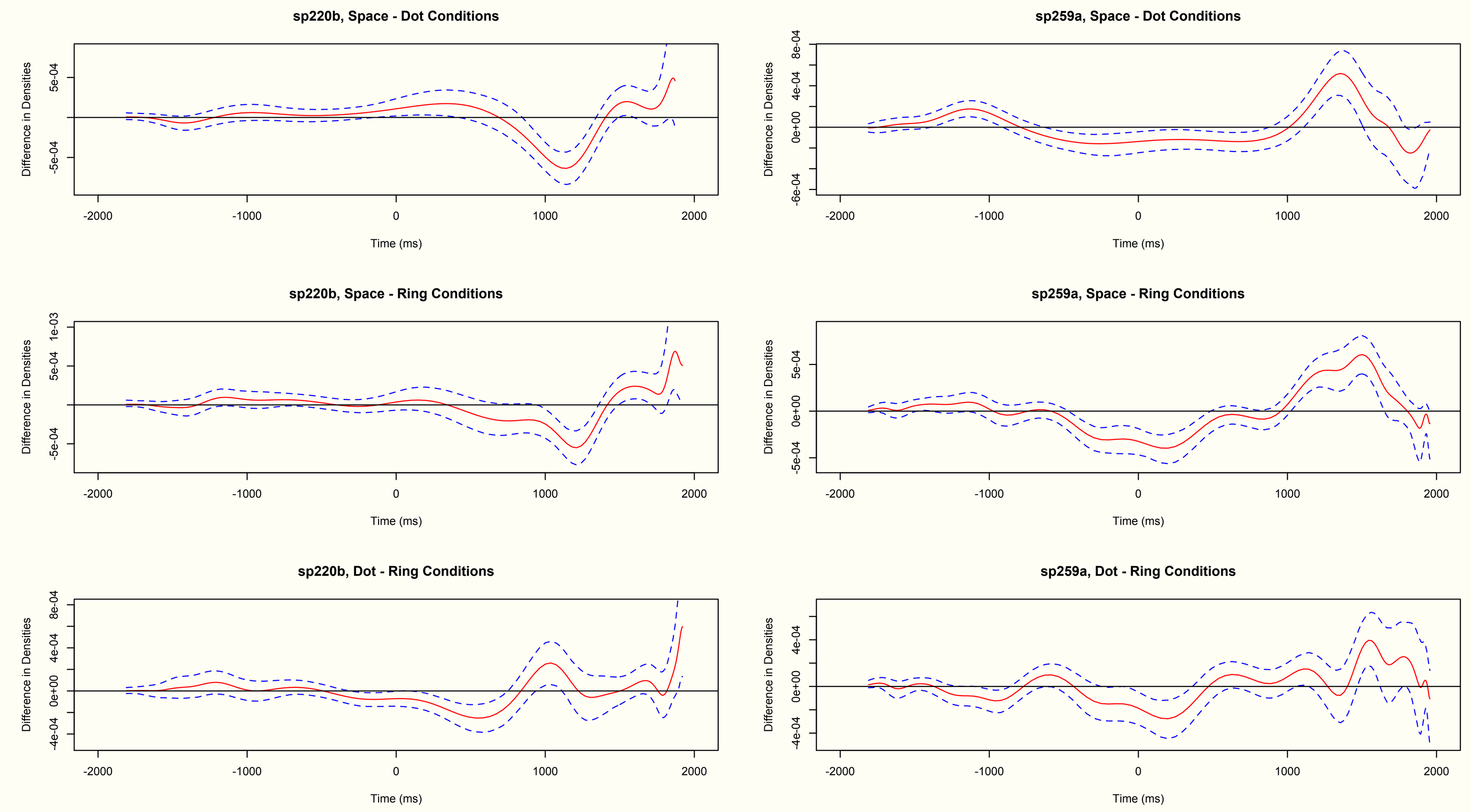
Results

We obtain posterior samples to acquire inference for the densities $f_i(t)$, which when multiplied by the posterior samples of γ_i, $i = 1, 2, 3$, provide realizations from the intensity functions of each condition. We report point (solid) and 95% interval estimates (dashed) for each condition on the original scale in the plots below (a).



(a)

Point (solid) and 95% interval estimates (dashed) were also obtain for the differences in the densities over the entire time interval, see below (b). The horizontal line represents no difference between the normalized intensity function (inferred density).



(b)

Concluding Remarks

In (a), sp220b exibits modes around −1200ms across all conditions, and around 1200ms for Dot and Ring conditions. For sp259a the Space condition exhibits bursts of activity around −1000, 0, and 1200ms. The bursts around 0 and 1200ms appear to be consistent across all conditions, but at different frequencies. In (b), we can see that there are intervals of time in which the normalized intensity functions differ. For sp220b, the space condition is statistically less active compared to the other two conditions. For sp259a, there is a difference around 0ms between all three conditions with the Ring condition being the highest frequency, followed by Dot condition, and then the Space condition. The Space condition in statistically higher than the other two in a small interval around 1200ms.